Research Paper

MATHEMATICAL MODELLING DYNAMICS BEHAVIOR OF FAST BREEDER REACTOR

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This work deals with the study of the dynamic performance of Liquid Metal Fast Breeder Reactor (FBR). The study involved rigorous mathematical modelling of the FBR and the developments of software powerful enough to study both normal and heavy transients of the dynamic performances. Modelling of FBR was developed based on lumped parameters technique which assumes average system parameters over defined lumps. This approach gives moderate internal information of the system depending on the number of lumps used with reasonable accuracy at moderate computer time and cost. The FBR model includes the neutronics, core heat transfer and piping. The developed FBR core heat transfer model used in this study is flexible. Three coolant nodes are used for the fuel node to obtain good approximation of the heat transfer driving force between fuel ad coolant which is taken as the difference between the average fuel temperature and the average temperatures of each coolant node respectively. FBR has insufficient stability, and therefore, fast acting control system may be required. The results of this study showed that for the reactivity change in FBR, the nuclear power and temperature changes.

Keywords: Dynamic performance, Mathematical modelling, Lumped parameters, Steady state, Heavy transients condition, Coolant nodes

INTRODUCTION

The behavior of fast reactors is quite different in some respects from water moderated thermal reactors, with implications for safety. The main differences are their neutron dynamics and properties of the coolant (liquid sodium in the PFBR). The same initiating events occurring in both thermal and fast reactors could produce very different outcomes (Kumar Ashwin and Ramana, 2008).

Simulation of transient conditions and related incidents and malfunctions such as failure/ tripping of pumps, heat exchangers, malfunction of valves, control systems, etc., affecting the system performance by altering the normal operation have also been modeled . The transient

simulation helps in understanding the dynamic behavior of the plant in a time frame extending from few seconds to tens of seconds. Fundamentally the plant design safety limits are fixed by the design experts, based on the transient analysis study conducted extensively, taking into consideration the various interconnected system behavior (Jayanthi *et al.*, 2013 and 2016).

The ultimate goal is to safeguard the plant and the personnel under all normal and abnormal conditions of the plant. The dynamic simulation study using training simulator can provide the operators, an opportunity to understand the system dynamics, the equipment performance and changes that would occur in the system with indicative parameters like pressure, level, temperature, flow, etc., with respect to time. The operator can understand the system behavior and subsequent level of stabilization after being subjected to a disturbance like pump trip or pump seizure, etc., and visualize and gain more insight about the plant dynamics. This will help the operator to make quick and accurate decisions towards improving the performance and safety of the plant (Jayanthi *et al.*, 2010 and 2014) generated in the core due to fission by fast neutrons. It then enters the IHX and flows downwards on the shell side. In FBR system, since the sodium is strongly activated, special precautions must be taken to contain sodium which may leak out of primary loop and to prevent its contact with atmosphere or water (sodium reacts violently with water and actively with air). Figure 1 is a schematic diagram of the FBR Reactor.In this study, a lumped parameter mathematical model is presented to simulate the reactor. The lumped parameter model assumes all system parameters to be averaged over defined regions (lumps) (Gaber, 1984). The reactor is represented by three axial regions, the lower plenum, the core and the upper plenum, the core is represented by an average channel in which the central fuel is surrounded by the primary sodium coolant, three coolant lumps are used for the fuel lumps (Hetrick, 1971; and Clinch River Breeder Reactor Plant Reference Design Report, 1974).

REACTOR MODEL

The reactor is represented by three axial regions, the inlet plenum, the core and the upper plenum as in Figure 1.


Figure 1: Schematic Diagram of the FBR Reactor
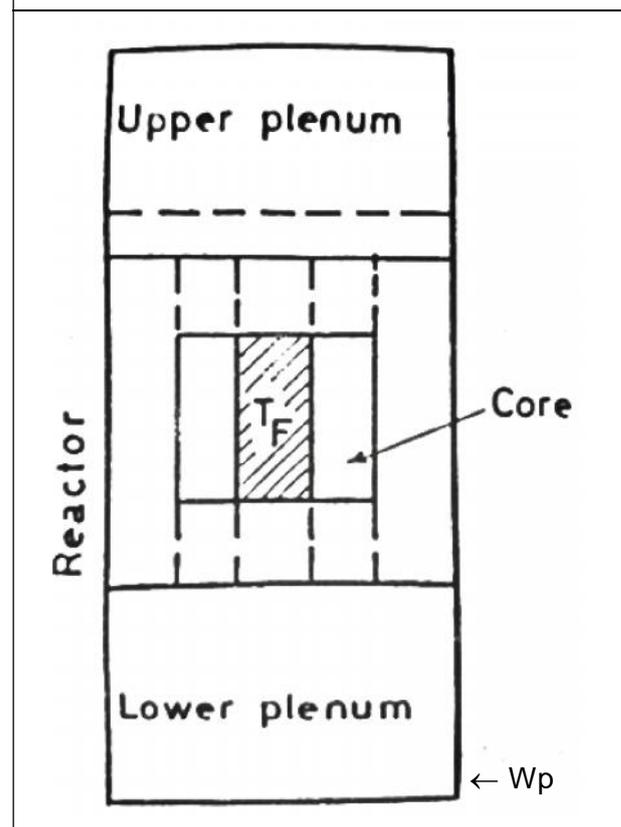
Reactor Thermal Hydraulics

In this section the governing equations for the core, lower plenum and upper plenum are developed.

Core Thermal Model

The model developed in this study is a simplified core model with one fuel lump and three coolant lumps in Figure 2. Three well stirred coolant nodes are used for each fuel node to obtain good approximation of the heat transfer driving force between fuel and coolant, the average temperature of coolant node is assumed to be equal to arithmetic mean of the inlet and outlet coolant node temperatures and the driving forces between fuel and coolant nodes (Q_1, Q_2, and Q_3) is taken as the difference between the average fuel temperature and the average temperature of the first, second and third coolant nodes respectively (Lamarch and Baratta, 2001; and Weston Sacey, 2007).

Applying the energy conservation equations on each of these four nodes, the following sets of equations are obtained:

Figure 2: Schematic Diagram of Reactor Thermal Model Lumps
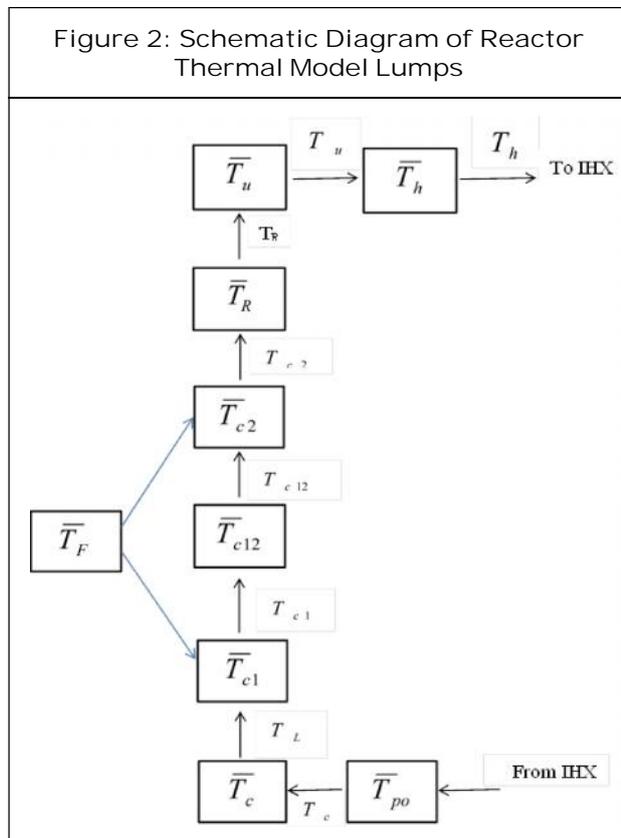
For fuel node:

$$M_F \ C_{PF} \frac{d}{dt}(T_F) = P_O \ f_p \ N - (Q_1 + Q_2 + Q_3) \quad (1)$$

For coolant nodes:

$$\frac{V_c}{3} S_{Pl}(\bar{T}_{c1}) C_{PP} \frac{d}{dt}(T_{c1}) = Q_1 + C_{PP} \ w_P \ ff \ (T_L - T_{c1}) \quad (2)$$

$$\frac{V_c}{3} S_{Pl}(\bar{T}_{c12}) C_{PP} \frac{d}{dt}(T_{c12}) = Q_2 + C_{PP} \ w_P \ ff \ (T_{c1} - T_{c12}) \quad (3)$$

$$\frac{V_c}{3} S_{Pl}(\bar{T}_{c2}) C_{PP} \frac{d}{dt}(T_{c2}) = Q_3 + C_{PP} \ w_P \ ff \ (T_{c12} - T_{c2}) \quad (4)$$

and

$$Q_1 = \frac{1}{3} U_{fc} . A_{fc}(T_F - \bar{T}_{c1})$$

$$Q_2 = \frac{1}{3} U_{fc} . A_{fc}(T_F - \bar{T}_{c12})$$

$$Q_3 = \frac{1}{3} U_{fc} . A_{fc}(T_F - \bar{T}_{c2})$$

where:

P_O = The total reactor power at normal operation, BTU/hr.

f_p = fraction of power generated in the core.

ff = fraction of flow into the core.

T_{c1}, T_{c12}, T_{c2} = output coolant temperature of first, second and third coolant nodes respectively, ^{O}F.

U_{fc} = overall heat transfer coefficient between fuel and coolant, BTU/hr. ft^2. ^{O}F.

A_{fc} = heat transfer area between fuel and coolant ft^2.

T_L = temperature of coolant entering the reactor core, ^{O}F.

C_{PP} = specific heat of the primary sodium, BTU/lb. ^{O}F.

C_{PF} = special heat of fuel, BTU/lb. ^{O}F.

T_F = average fuel temperature, °F.

w_P = primary sodium coolant flow rate, lb/hr.

M_F = Mass of fuel, lb.

V_c = volume of coolant in the reactor core, ft³.

$S_{PI}(\bar{T}_{c1})$, $S_{PI}(\bar{T}_{c12})$, $S_{PI}(\bar{T}_{c2})$ = average coolant density in the first, second and third coolant nodes respectively, lb/ft³.

$$\bar{T}_{c1} = (T_L + T_{c1})/2$$

$$\bar{T}_{c12} = (T_{c1} + T_{c12})/2$$

$$\bar{T}_{c2} = (T_{c12} + T_{c2})/2$$

Lower Plenum

Figure 2 shows a schematic diagram of reactor thermal model lumps. In both lower and upper plena complete mixing is assumed during normal transients. Applying the energy equation for the coolant in the lower plenum, the following equation is obtained:

$$S_{PI}(\bar{T}_c)V_L \frac{d}{dt}(T_L) = W_P(T_c - T_L) \quad (5)$$

where:

V_L = volume of primary sodium coolant in the lower plenum, ft³.

T_c = temperature of the reactor coolant entering the lower plenum, °F.

$S_{PI}(\bar{T}_c)$ = average coolant density in the lower plenum, lb/ft³.

$$\bar{T}_c = (T_c - T_L)/2$$

Upper Plenum

As in the lower plenum, we can apply the energy equation for the coolant with T_R representing the average (mixed) temperature /3/ at the entrance of upper plenum. T_R is obtained from the equation:

$$C_{PP}V_{NA}(S_{PI}(T_R) + S_{PI}(T_L))/2 * \frac{d}{dt}(T_R) = P_O\,N - C_{PP}W_P(T_R - T_L)$$
$$...(6)$$

$$S_{PI}(\bar{T}_U)V_U \frac{d}{dt}(T_U) = W_P(T_R - T_U) \quad ...(7)$$

where:

$S_{PI}(T_R)$, $S_{PI}(T_L)$, $S_{PI}(\bar{T}_U)$ = sodium density at temperature (T_R), (T_L) and $(\bar{T}_U)$ respectively, lb/ft³

V_{NA} = volume of sodium in the reactor vessel less than in the lower and upper Plenum/16/, ft³.

T_U = coolant temperature in the upper plenum, °F.

V_U = volume of coolant in the upper plenum, ft³.

$$= (T_U + T_R)/2 \quad \bar{T}_U = °F$$

For the application of mathematical model, it is necessary to obtain the effective heat transfer coefficient between the primary sodium coolant with a bulk temperature T_b and the fuel with an average temperature T_F. This heat transfer coefficient can be derived by equating the heat generated in the fuel rods with the heat transferred by conduction through the cladding material and the heat transferred by convection through the coolant film. It is assumed that no heat generated in the cladding or the coolant and the resistance to heat flow at the fuel cladding interface is neglected (Rouben, 2015).

The overall heat transfer coefficient between fuel and coolant (BTU/hr.ft². °F) is:

$$U_{fc} = 1/\left(\frac{1}{h_p}\frac{D_F}{D_{C1}} + \frac{\ln\frac{D_{C1}}{D_F}}{2K_{Cl}}D_F + \frac{1}{8K_F}D_F\right) \quad ...(8)$$

where:

h_p = the primary film heat transfer coefficient= (BTU/hr.ft². °F).

D_F = diameter of fuel rod, ft.

$D_{c1} = D_F + 2c$, where c is the cladding thickness in ft.

K_{cl} = thermal conductivity of cladding material (BTU/hr.ft^2. $^{\circ}$F).

K_F = thermal conductivity of fuel (BTU/hr.ft^2. $^{\circ}$F).

The heat transferred from fuel to coolant is given by the following equation (El-Wakil, 1982):

Table 1: Main Data Related to 380 MWe	
Parameters	**Values**
1. Overall Plant	
Rated thermal power	975 MW
Heat generated in fuel core	92.39%
Net electrical output power	380 MW
2. Reactor	
a. Fuel Assembly	
Number of fuel rods	42966
Fuel rod	
Length	3 ft
Outside diameter	0.23 inch
Cladding material	SS-316
Cladding thickness	0.03 inch
Inside diameter	0.2 inch
Pitch	0.2875
Triangular pitch to diameter ratio	1.25
Sodium equivalent area fuel pin	2.0857*10-4ft
Equivalent (hydraulic) diameter	0.013855 ft
Fraction of the flow into core	0.8
b. Coolant	
Temperature: Vessel inlet	730 $^{\circ}$F
Vessel outlet	994.73 $^{\circ}$F
Flow rate	41.446*106Ib/hr

$$Q = U_{fc} A_{fc} (T_F - T_b) \qquad ...(9)$$

Substituting for U_{fc} from Equation (8) in Equation (9) and using the design values for Q and A_{fc} given in Table 1, Equation (9) can be solved for T_F to get the steady state fuel temperature.

Reactor Kinetic

The point-kinetic equation with prompt jump approximation and six delayed neutron groups is utilized to describe the neutron balance in the equivalent core. This equation can be written in the form:

$$\frac{dn}{dto} = (R_T - B_T)n + \Sigma_{i=1}^{6} \lambda_i \, C_i \Lambda \qquad ...(10)$$

$$\frac{dCi}{dt} = \frac{Bi}{\Lambda} n - \lambda_i C_i \qquad i = 1,2,\ldots\ldots\ldots6 \qquad ...(11)$$

where:

n = Neutron density, neutron/cm^3.

B_T = total delayed neutron fraction.

Λ = neutron generation time, s.

B_i = delayed neutron fractionfor the delayed neutron group.

λ_i = decay constant for ith delayed neutron group, s^{-1}.

C_i = ithprecursor concentration neutron/cm^3.

$R_T = R_{fb} + R$.

R = respectively insertion caused by control rod movement

R_{fb} = feedback respectively, which satisfied the following equation:

$$R_{fb} = \left(100 \times DC/B_T\right) \ln \frac{460 + T_F}{460 + T_I} + AX(T_F - T_I) + SD(\bar{T}_L - T_I) +$$

$$RE(T_L - T_I) \qquad ...(12)$$

for $\bar{T}_L, T_L \geq T_I T_F$,,

where:

DC = Doppler constant, $(T_F \frac{dK}{dT_F})$.

T_F = average fuel temperature $^\circ$F.

T_L = temperature of coolant entering the reactor core $^\circ$F.

T_I = cold stand by temperature (400°F).

AX = axial expansion reactivity coefficient (Φ/$^\circ$F).

SD = sodium density reactivity coefficient (Φ/$^\circ$F).

RE = radial expansion reactivity coefficient (Φ/$^\circ$F).

$= (T_L + T_{c2})/2 \bar{T}_L =$ $^\circ$F.

T_{c2} = Output coolant temperature of the core $^\circ$F.

The reactor design parameters necessary for the steady state and dynamic response calculations are given in Table 2.

Primary Coolant Transport in Piping

In this model, the equations describing temperatures of the reactor coolant in the hot and cold leg piping are:

$$M_h \frac{d}{dt}(T_h) = W_P(T_U - T_h) \quad ...(13)$$

$$M_C \frac{d}{dt}(T_C) = W_P(T_{PO} - T_C) \quad ...(14)$$

where:

T_h = primary coolant temperature in the hot leg piping, $^\circ$F.

T_{pO} = primary coolant temperature leaving IHX, $^\circ$F.

Table 2: Reactor Design Parameters for 380 MWe

Reactor Parameters	Values
MU-238	5065 kg
MU-235	35.5 kg in Uo2
MPU	1475 kg in PUO2
MF	6576.4 kg
VU	6325 ft3
VNA	4928 ft3
VL	3000 ft3
CPE	0.8 BTU/Ibm.Fo
CPP0	3032 BTU/Ibm.Fo
CPI0	30442 BTU/Ibm.Fo
KF	1.4063 BTU/Ibm.ft. OF
KCL	12.4 BTU/Ibm.ft. OF
Ufc.Afc	2303457.6 BTU/hr. OF
BT	3.65*10-3
B1	0.8254*10-4
B2	7.766*10-4
B3	6.660*10-4
B4	13.54*10-4
B5	1.8*10-4
λ1	.0129 sec-1
λ2	0.312 sec-1
λ3	0.1330 sec-1
λ4	0.3450 sec-1
λ5	1.4100 sec-1
λ6	3.75 sec-1
DC	-0.0058 Φ/OF
AX	-0.023ϕ/OF
SD	-0.003 ϕ/OF
RE	-0.21ϕ/OF
Rfb	-2.9444 $
R	+2.9444$

M_C = mass of coolant in the cold leg ping, lb.

M_h = mass of coolant in thehotleg piping, lb.

STEADY STATE CALCULATIONS

The developed models of reactor consists of a group of first order differential equations and algebraic equations, which can be solved by a numerical method called "MERSON". This method consists of main program and two subroutines written in Fortranlanguage.The main program contains the input data, the first subroutine contains the method of solution and the second subroutine contains the differential equations in the form of: $\frac{d}{dt}(\overline{Y}) = F(\overline{Y})$, where $\overline{Y}$ is the vector of state variable chosen as listed in Table 3.

The steady state values were obtained from the state equations with the time derivation equal to zero. The obtained steady state values of the reactor are presented in Table 4.

THE RESULTS

This section presents a study of the results of

Table 4: Steady State Results for FBR Reactor (380 MWe)	
Reactor Parameters	**Values**
N	1
P_O	975 MW
T_F	2217.18 ^{O}F
T_{C1}, T_{C12}, T_{C2}	840,942.5,1035.73 ^{O}F
T_U	994.73 ^{O}F
T_L	730 ^{O}F
T_h	994.73 ^{O}F
T_{pi}	994 ^{O}F
T_{po}	730 ^{O}F

the dynamic response calculations obtained from the mathematical model developed in this study. The transient responses of Fast Breeder Reactor (FBR) for a step change of ± 30 μ in the external reactivity as shown in Figure 3.

The first effect of the –ve reactivity insertion appears as a rapid decreases in power generated due to the external reactivity decreases and the use of the prompt jump approximation in evaluating the neutron density in the reactor. The reason that the coolant temperatures in the core initially decreases at a rate not much slower than that of the fuel temperature is that the mass heat capacity (M_{Cp}) of the fuel in the core is larger than that of the coolant.

The continuous decreases in the average and exit coolant temperatures of the core, even after the neutron power density and fuel temperature level off, is due to the continuous decrease in the coolant inlet temperature to the core. Positive 30 μ reactivity insertion affects the same parameters as explained in the case of negative reactivity insertions but in an opposite way.

Table 3: State Variables Index of FBR Reactor		
Number	**Symbol**	**Description**
1	N	Normalized neutron density (n/no)
2-7	C_1-C_6	precursor concentration of group, I = 1, …, 6
8	T_F	Fuel temperature
9-11	T_{C1}, T_{C12}, T_{C2}	Coolant output temperature of the first, second and third lumps
12	T_U	Reactor upper plenum output temperature
13	T_R	Reactoraverage mixed temperature at the entrance of the upper plenum
14	T_L	Reactor lower plenum output temperature
15	T_C	Reactor lower plenum input temperature
16	T_h	Reactor hot leg temperature

Figure 3a: The Effect of the Change in the External Reactivity on the Normalized Neutron Density (n/no) and the Fuel Temperature
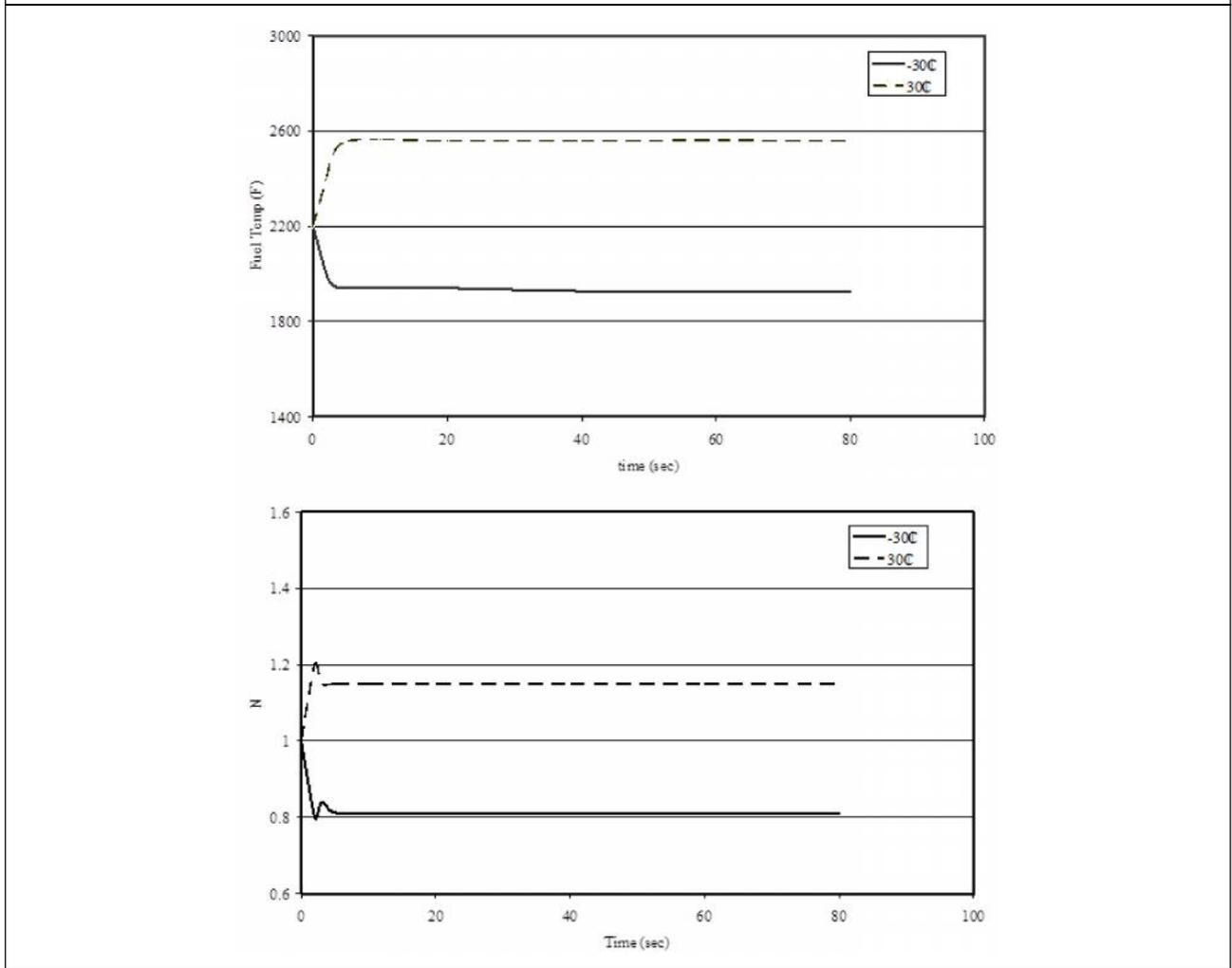

Figure 3b: Plant Transient Responses or FPR System as Shown in T_R and T_L for a+ of -30 ℃ Step Change in the External Reactivity
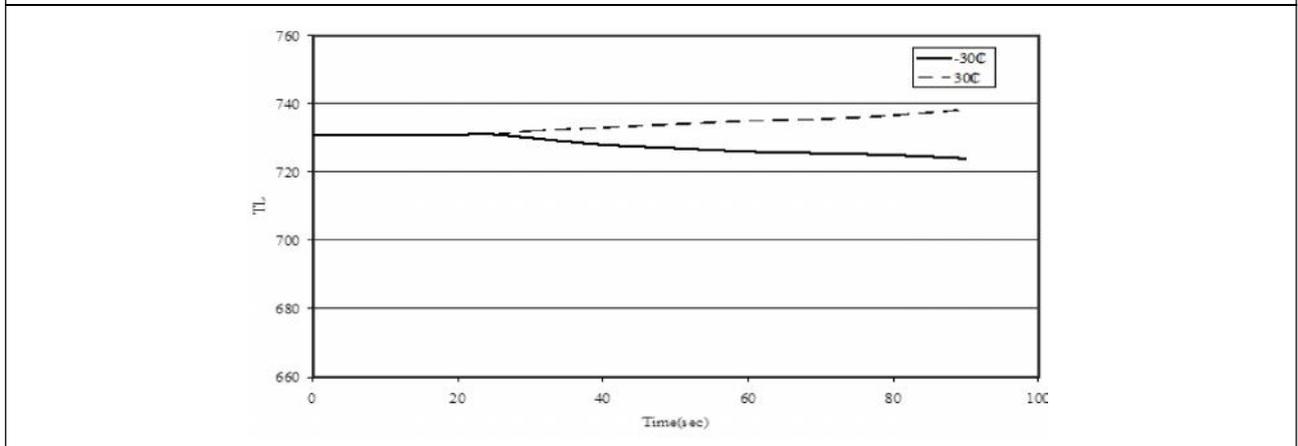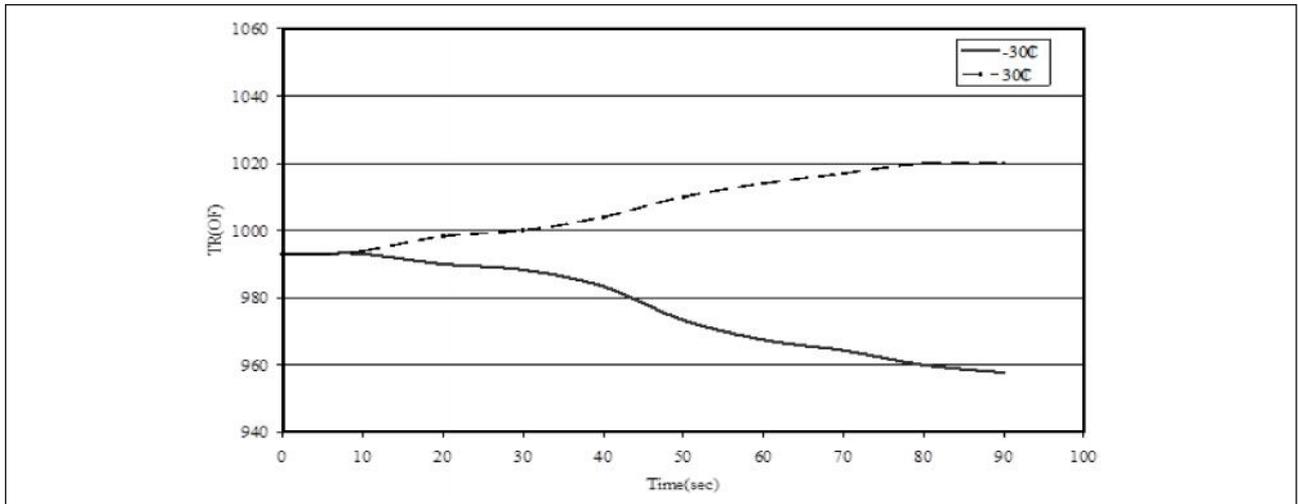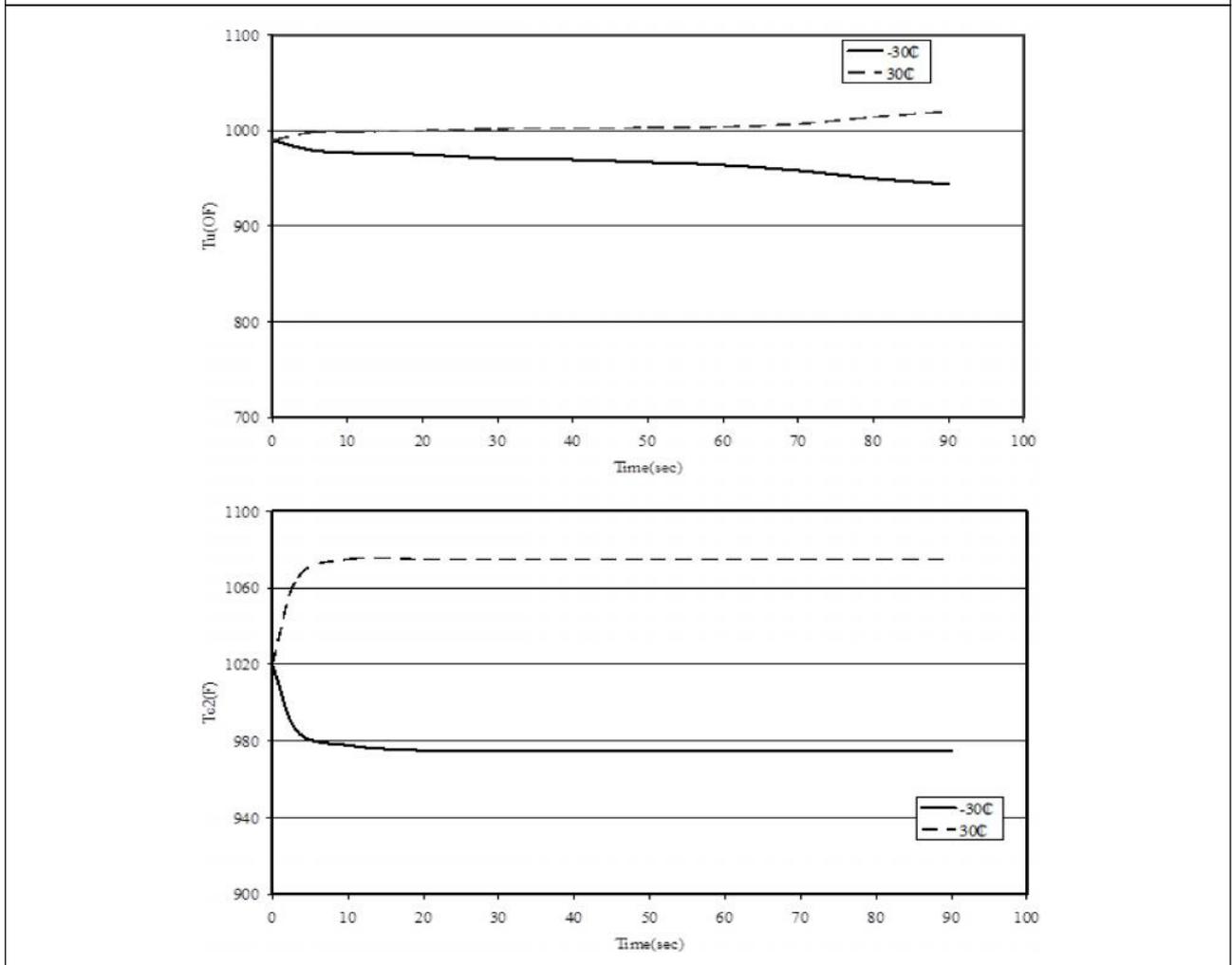
Figure 3b (Cont.)

Figure 3c: Plant Transient Responses or FPR System as Shown in T_{c2} and T_u for a+ of -30 ℃ Step Change in the External Reactivity

CONCLUSION

In this work a mathematical model for FBR reactor has been developed. The FBR lumped parameter model includes the neutronics, core heat transfer and piping. This study involved the developments of Fortrancode to study the FBR dynamic performances during different values of perturbation, FBR is stable at small changes in reactivity. This can be interpreted as the large negative temperature coefficient in FBR. FBR does not return near to its original state, because it is inherently unstable, and fast acting control system may be required.

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