PDP ESTIMATION USING LINEAR MINIMUM MEAN SQUARE ERROR IN MIMO-OFDM SYSTEMS

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Over the recent years, the combination of Multiple Input Multiple Output (MIMO) with Orthogonal Frequency Division Multiplexing (OFDM) has gained significant interest and is considered as one of the most promising techniques for present and future wireless communication systems such as Wi-Max (IEEE 802.16) or 3rd Generation Partnership Project Long Term Evolution (3GPP LTE). In Linear Minimum Mean Square Error (LMMSE) channel estimation for multicarrier system, it is necessary to know channel correlation function. Hence the estimation of noise variance using null carrier and Power Delay Profile (PDP) can be used to approximate which can be described in two parameters like; mean delay and Root Mean Square (RMS) delay spread. In this paper the estimation of channel at pilot frequencies with LMMSE estimation algorithms is carried out through Mat lab simulation. Simulation results show that the performance of LMMSE channel estimation using the proposed PDP estimate approaches that of Wiener filtering due to the mitigation of distortion effects.

Keywords: Power delay profile, MIMO, OFDM, 3 GPP-LTE, RMS, Wiener filtering

INTRODUCTION

Many wireless communications systems, the combination of Multiple Input Multiple Output (MIMO) with Orthogonal Frequency Division Multiplexing (OFDM) has gained significant interest and is considered as one of the most promising techniques for present and future wireless communication systems such as Wi-Max or 3G. The optimal training sequences and pilot tones for Orthogonal Frequency Division Multiplexing (OFDM) channel estimation were investigated in MIMO-OFDM Channel Estimation using Pilot Carries, and many more papers have proposed different problems in MIMOs. In this paper, a major focus is on the Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi carrier modulation scheme, which uses a large number of closely spaced orthogonal sub-carriers. A single stream of data is split into parallel streams each of which is coded and modulated on to a subcarrier, a term commonly used in OFDM systems. Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation) at a low symbol...
rate, maintaining data rates similar to conventional single carrier modulation schemes in the same bandwidth. Thus the high bit rates seen before on a single carrier is reduced to lower bit rates on the subcarrier. In practice, OFDM signals are generated and detected using the Fast Fourier Transform algorithm.

In OFDM, subcarriers overlap, they are orthogonal because the peak of one subcarrier occurs when other subcarriers are at zero. This is achieved by realizing all the subcarriers together using Inverse Fast Fourier Transform (IFFT). Note that each subcarrier can still be modulated independently. Once orthogonality is lost we experience Inter-Carrier Interference (ICI). This is the interference from one subcarrier to another. There is another reason for ICI. Adding the guard time with no transmission causes problems for IFFT and FFT, which results in ICI. A delayed version of one subcarrier can interfere with another subcarrier in the next symbol period. This is avoided by extending the symbol into the guard period that precedes it. This is known as a cyclic prefix. It ensures that delayed symbols will have integer number of cycles within the FFT integration interval. This removes ICI so long as the delay spread is less than the guard period.

The aim of this paper is to investigate the OFDM scheme, and to reduce the mismatch in the frequency domain, we propose a PDP estimation technique for the LMMSE channel estimator of MIMO-OFDM systems. Simulation results show that the performance of LMMSE channel estimation with the proposed PDP estimate approaches that of Wiener filtering.

CHANNEL ESTIMATION METHODS IN OFDM SYSTEMS

Channel estimation plays a very important role in OFDM system. Many related algorithms have been presented these years, which can be generally separated into two methods, pilot-based channel estimation and blind channel estimation, pilot-based channel estimation is a practical and an effective method.

Blind Channel Estimation

Blind channel estimation, this uses statistical information of the received signals. Blind channel estimation methods avoid the use of pilots and have higher spectral efficiency. However, they often suffer from high computation complexity and low convergence speed since they often need a large amount of receiving data to obtain some statistical information such as cyclostationarity induced by the cyclic prefix. Therefore, methods are not suitable for most practical communication systems such as World Interoperability for Microwave Access (WIMAX) system adopt pilot assisted channel estimation.

Pilot Channel Estimation

Pilot-based channel estimation estimates the channel information by obtaining the impulse response from all sub-carriers by pilot. Pilot-based channel estimation is based on the transmission of symbols that are known to receiver so called pilot symbols. The pilot symbols are inserted into data stream and transmitted over mobile channel, at receiver pilot symbols are analyzed in order to obtain channel estimate which is utilized for equalization. As the characteristic of mobile channel is varying with time and frequency in order to obtain estimate which provides information about time and frequency domain channel variation, pilot symbol need to be transmitted periodically in time and spread over whole bandwidth which is provided for data transmission. For the pilot-aided channel estimation methods, there are two classical pilot
patterns, which are the block-type pattern and the comb-type pattern. The block-type refers to that the pilots are inserted into all the subcarriers of one OFDM symbol with a certain period, i.e., symbols are transmitted periodically, and all subcarriers are used as pilots. The block-type can be adopted in slow fading channel, that is, the channel is stationary within a certain period of OFDM symbols. The comb-type refers to that the pilots are inserted at some specific subcarriers in each OFDM symbol. The comb-type is preferable in fast varying fading channels, that is, the channel varies over two adjacent OFDM symbols but remains stationary within one OFDM symbol. The comb-type pilot arrangement-based channel estimation has been shown as more applicable since it can track fast varying fading channels, compared with the block-type one.

Figure 1: Pilot Based OFDM System Model

SYSTEM MODEL

The Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique, which divides the available spectrum into many carriers, each one being modulated by a low rate data stream. OFDM overcomes most of the problems with both FDMA and TDMA. OFDM splits the available bandwidth into many narrow band channels (typically 100-8000). The carriers for each channel are made orthogonal to one another, allowing them to be spaced very close together, with no overhead. Because of this there is no great need for users to be time multiplex as in TDMA, thus there is no overhead associated with switching between users. OFDM is a special form of MCM and the OFDM time domain waveforms are chosen such that mutual orthogonality is ensured even though sub-carrier spectra may over-lap. The sinc function, illustrated in Figure 2 exhibits this property and it is used as a carrier in an OFDM system.

Let us consider a MIMO-OFDM system, with \( p \) transmits and \( q \) receive antennas, and \( t \) total subcarriers.

Suppose that the MIMO-OFDM system with the specified antennas transmits \( k_d \) subcarriers at the central spectrum assigned for data and pilots, in order to control interferences with other systems. Let \( [k_p, n_p] \) be the pilot subcarrier for the \( p \)th transmit antenna at the \( n_p \)th OFDM symbol, which is a QPSK modulated signal. We assume that the pilot subcarriers are distributed over a time and frequency grid as in

At the \( n_p \)th OFDM symbol, the number of pilot subcarriers is defined as \( k_p = p \). The pilot inserted

Figure 2: To Preserve the Orthogonality of Pilots Among Different Transmits Antennas

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OFDM symbol shown in Figure 3, is transmitted over the wireless channel after performing an Inverse Fast Fourier Transform (IFFT) and adding a CP. It is assumed that the length of CP, $L_g$, is longer than the channel maximum delay, $L_{ch}$, making the channel matrix circulate ($L_{ch} \leq L_g$). At the receiver, after perfect synchronization, the removal of CP, and FFT operation, the received pilot symbol for the $q^{th}$ receive antenna can be represented as,

$$Y_q[n_p] = \text{Diag}(X_p)F_p,h_{p,q} + n_p \quad \text{...(1)}$$

where $h_{p,q} = [h_{p,q}^1(n_p, 0), h_{p,q}^1(n_p, 1), \ldots, h_{p,q}^1(n_p, L_{ch}), 0]^T$ is an $L_g \times 1$ CIR vector at the $p^{th}$ transmit antenna and $q^{th}$ receive antenna.

From (1), the CIR at the $(p, q)^{th}$ antenna port can be estimated approximately using the Regularized Least Squares (RLS) channel estimation with a fixed length of $L_g$ as:

$$\hat{h}_{R,p,q} = (F_p^H F_p + \varepsilon I_L)^{-1} F_p^H \text{ diag}(X_p)^H Y_q[n_p] \quad \text{...(2)}$$

$$= W_{RLS,p} Y_q[n_p] \quad \text{...(3)}$$

where $\varepsilon = 0.001$ is a small regularization parameter, and $I_L$ is the $L_g \times L_g$ identity matrix. To derive the PDP from the estimated CIR in (2), the ensemble average of $h^H R, h^H H$ is given by

$$E[h_{R,p,q}^H h_{R,p,q}] = WR_h W^H + \sigma_w^2 W_{RLS,p} W_{RLS,p}^H \quad \text{...(4)}$$

where

$$R_{in} = E(h_{p,q}^H h_{p,q}^t) \quad \text{and} \quad W = (F_p^H + I_{L_g})^{-1} F_p^H F_p$$

$R_{hh}$, represent the PDP of multipath channel within the length of $L_g$. Unfortunately, $R_{hh}$ is distorted by $W$, which is an ill-conditioned matrix due to the presence of $F_p^H F_p$. Thus, instead of calculating $W^{-1}$, we investigate the method for eliminating the spectral leakage of $W$. The covariance matrix of the estimated CIR is defined as $R_{in} = WR_{hh} W_r$, which can be expressed as

$$R_{in} = \sum_{l=0}^{L_g-1} W \text{diag}(u_l) W^H \quad \text{...(5)}$$

where $u_l$ is a unit vector with the $l^{th}$ entry being one and otherwise zeros. Let $P_{g}$ and $t_l$ be the $L_g \times 1$ vectors defined as $P_l = (R_{h_l})$ and $t_l = D_g (W \text{diag}(u_l) WH)$, respectively, where $D_g(A)$ is the column vector containing all the diagonal elements of $A$. Then, the relation in (4) is simplified as

$$P_l = p_l t_0 + p_l t_1 + \ldots + p_{L_g} t_{L_g-1} = TP_l \quad \text{...(6)}$$

where $T = [t_0, t_1, \ldots, t_{L_g-1}]$ is defined as a distortion matrix by $W$. In addition, the distortion matrix is a well-conditioned matrix. Hence, the distortion of $W$ can be eliminated as:

$$P_l = T^{-1} P_l = E[g_{p,q}^H[n_p]] - \sigma^2 \tilde{w} \quad \text{...(7)}$$

where

$$g_{p,q}^H[n_p] = T^{-1} D_g (\hat{h}_{R,p,q} h_{R,p,q})$$

is defined as the received sample vector for estimating PDP at the $(p, q)^{th}$ antenna port on the $r^{th}$ OFDM symbol, and

$$\tilde{w} = T^{-1} (W_{RLS,p} W_{RLS,p}^H) \quad \text{...(8)}$$

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The received sample vector in (7) can be expressed as

\[ G_{p,q}[n] = D_g(h_{p,q}h_{p,q}^H) + \tilde{h}_{p,q} + e_{p,q} \quad \text{...(9)} \]

where \( \tilde{h}_{p,q} = T^{-1}D_g(W_{RLS,p}n_qh_q^HW_{RLS,p}^H) \) and

\[ e_{p,q} = 2R_e[T^{-1}(Wh_{p,q}n_q^HW_{RLS,p}^H)] \]

Here, \( Re(a) \) denotes the real part of \( a \). We assume that \( \tilde{h}_{p,q} \) is an effective noise by AWGN. Then, the sample average of \( g_{p,q}[n] \) is given by

\[
< g_{p,q}[n] > N \approx \frac{1}{N} \sum_{n=1}^{N_p} \sum_{p=1}^{P} \sum_{q=1}^{Q} g_{p,q}[n]
\]

\[
= (D_g(h_{p,q}h_{p,q}^H)) + (\tilde{h}_{p,q})N + (e_{p,q})N
\]

\[
= \langle D_g(h_{p,q}h_{p,q}^H) \rangle + \langle \tilde{h}_{p,q} \rangle N + \langle e_{p,q} \rangle N \quad \text{...(10)}
\]

where \( N \approx [\tau_p]PQ \) represents the total number of samples for PDP estimation. \( |\tau_p| \) is the number of pilot symbols at the \( k_p \)th subcarrier in a time slot. When is sufficiently large, the PDP can be perfectly estimated, since \( \langle D_g(h_{p,q}h_{p,q}^H) \rangle N \rightarrow Ph(\tilde{h}_{p,q}) \)

\[ N \rightarrow \sigma_p^2 \tilde{w} \] and \( \langle e_{p,q} \rangle N \rightarrow 0 \). However, it is difficult for a receiver of practical MIMO-OFDM systems to obtain such a large number of samples. With an insufficient number of samples, the PDP can be approximated as

\[
P_h \{ \approx D_g(h_{p,q}h_{p,q}^H) \} N \quad \text{...(12)}
\]

To improve the accuracy of PDP estimation with insufficient we mitigate the effective noise as follows

\[
< g_{p,q}[n] > N \approx \sigma_p^2 \tilde{w} = \langle D_g(h_{p,q}h_{p,q}^H) \rangle + Z_N \quad \text{...(13)}
\]

where

\[ Z_N = \langle e_{p,q} \rangle N + \langle \tilde{h}_{p,q} \rangle N \sigma_p^2 \tilde{w} \]

is defined as a residual noise vector, in which each entry has a zero-mean. Then, the error of PDP estimation with \( N \) samples can be calculated as

\[ e_N = \langle (D_g(h_{p,q}h_{p,q}^H) \rangle N - P_h \rangle + Z_N \quad \text{...(14)} \]

Since \( |P_h| \geq 0 \) for all \( i \), the PDP can initially be estimated as

\[ P_{inv} = \frac{1}{N} \sum_{n=1}^{N_p} \sum_{p=1}^{P} \sum_{q=1}^{Q} S_{p,q}[N_p] \quad \text{...(15)} \]

where \( S_{p,q}[N_p] \) is the sample vector of proposed PDP estimator with the \( l \)th entry

\[ S_{p,q}[N_p] = \begin{cases} g_{p,q}[N_p] - \sigma_p^2 \tilde{w} & \text{if } g_{p,q}[N_p] \geq \sigma_p^2 \tilde{w} \\ 0 & \text{otherwise} \end{cases} \quad \text{...(16)} \]

where \( g_{p,q}[N_p] = [g_{p,q}[N_p]] \) and \( \tilde{w} = [\tilde{w}] \). To mitigate the detrimental effect of residual noise \( Z_N \) the proposed scheme estimates the average of residual noise at the zero-taps of \( P_h \). At the \( l \)th entry of, \( \hat{P}_{inv} \) the zero-tap can be detected as

\[ t_z = \begin{cases} 1 & \text{if } \hat{P}_{inv} < \beta \theta \\ 0 & \text{otherwise} \end{cases} \]

where

\[ \beta \theta = \frac{1}{L_y} \sum_{i=1}^{L_y} \hat{P}_{inv} \]

is defined as a threshold value for the zero-tap detection. Then, the average of residual noise at the zero-taps can be estimated as

\[ \hat{n}R_{avg} = \frac{1}{N_y} \sum_{i=1}^{L_y} \hat{P}_{inv} t_z \quad \text{...(17)} \]
where \( N_z = \sum_{n=0}^{L_g-1} t^l \) represents the total number of detected zero-taps. With the mitigation of residual noise, the \( l \)th tap of the PDP estimate, \( \hat{\rho}_n \), can be expressed as

\[
\hat{\rho}^l_n = \begin{cases} 
\hat{\rho}^l_{tot} - \hat{\rho}^l_{avg} & \text{if } \hat{\rho}^l_{tot} > \hat{\rho}^l_{avg} \\
0 & \text{otherwise}
\end{cases} \quad \ldots (18)
\]

Then, the estimated PDP in above equation can be used to obtain the frequency-domain channel correlation in the LMMSE channel estimator.

**SIMULATION RESULTS**

We consider a MIMO-OFDM system. The system bandwidth is 5 MHz with 301 subcarriers for transmitting data information and pilots at 2 GHz carrier frequency. Number of pilots are 12. One frame consist of 14 OFDM symbols. The MIMO-OFDM system utilizes two transmit and one receive antennas \((P = 2, Q = 1)\). The length of CP is 40 \((L_g = 40)\). One important parameter of the channel is the power delay profile \( R_{hh} \) which represents the average power (also called multipath intensity profile) associated with a given multipath delay.

In Figure 4, we investigate the MSE performance of the proposed scheme over the exponentially power decaying six-path Rayleigh fading channel model, where the channel maximum delay, \( L_{ch} \), is variable. The performance of the proposed scheme is better than that of the conventional methods, and approaches that of Wiener filtering in various channel environments.

Figure 5, it can be seen that the MSE of LMMSE technique using the estimated PDP Figure 4: Performance of LMMSE Technique with Variable Channel Length. \( L_{ch} = 35 \)
Figure 5: Performance of LMMSE Technique Using the Estimated PDP with Different mobile Equipment Speeds (Km/hr)

Figure 6: Simulation and Analysis Results of LMMSE Channel Estimation
achieves that of Wiener filtering even at high Doppler frequencies.

Figure 6 shows simulation and analysis results of the frequency-domain LMMSE channel estimation with various samples for obtaining the PDP.

The simulation results correspond to the channel estimation performance at the first OFDM symbol of antenna port 1.

In Figure 6 it is observed that the MSE of the proposed scheme improves the MSE performance with an increase in the number of samples for PDP estimation.

And Figure 7 shows the performance of LMMSE and SNR.

**CONCLUSION**

In this paper, we proposed an improved power delay profile estimation in multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) systems under linear Minimum Mean Square Error (LMMSE) channel. The CIR estimates at each path of the MIMO channels were used to obtain the PDP. For proper PDP estimation, we considered the spectral leakage effect from virtual subcarriers, and the residual noise caused by the insufficient number of estimated CIR samples. The proposed technique effectively reduces both the spectrum leakage and residual noise. Simulation results show that the performance of LMMSE channel estimation using the proposed PDP estimate approaches that of Wiener filtering.

**REFERENCES**


