The paper develops a model for the deployment of optimized quantity of dumpers in a synchronized operation between hot mix asphalt plant production, transportation of hot mix asphalt and laying at pavement construction site. The queuing theory has been applied in arriving at the formula for calculation of quantity of dumpers. The results obtained have been validated by applying it to a runway construction project. The formula can prove to be quite handy in day to day planning for the site engineers and it can become an effective tool in eliminating the wastages. Also a relationship has been developed between plant production, the capacity of dumpers and the capacity of the laying site to handle the number of dumpers and time spent by dumpers at laying site. The relationship so developed can be useful in the selection of right type and capacity of Hot Mix Asphalt (HMA) plant, Paver Finishers and Dumpers.

**Keywords:** Hot Mix Asphalt (HMA), Flexible Pavement, Plant site, Laying Site, Paver Finisher, Dumper, Operational Synchronization, Queuing Theory, Poisson's Distribution, Exponential Distribution

**INTRODUCTION**

The construction of flexible pavements consists of two main working sites, viz., production facility at hot mix asphalt plant and asphalt laying site. The production of hot mix asphalt and laying works are highly mechanized in nature and require great deal of operational synchronization. The Hot Mix Asphalt (HMA) plant production is similar to manufacturing industries in which large quantities of high quality, products are produced at the lowest possible cost (United Facilities Criteria, 2001). As with most manufacturing
facilities, raw materials must be shipped in (asphalt binder and aggregate), and finished products must be shipped out (HMA) (Pavement Interactive, 2012). However the HMA production and laying is marred by uncertainties and constraints mainly due to environment, traffic, etc. (Indiana Department of Transportation, 2012). The variable conditions can be taken care of by applying probabilistic distributions in production and transportation and the operation can be synchronized using a queuing model (Allen, 1990). The model so developed may be validated with the actual data obtained from the construction of flexible pavements of runway.

MATERIAL AND METHODS
Application of Queuing Theory to Achieve Synchronization in HMA Production

Mix transport involves all actions and equipment required to convey HMA from a production facility to a paving site including truck loading, weighing and ticketing, hauling to the paving site, dumping of the mix into the paver and truck return to the HMA production facility. Ideally, the goal of mix transport should be to maintain mix characteristics between the production facility and the paving site (Pavement Interactive, 2012).

The operation of hot mix asphalt plant and bitumen laying site needs to be coordinated closely as the there is constant flow of hot asphalt mix materials loaded in dumpers from plant site to the laying site. The dumpers once emptied in the laying site are returned to the plant site. The bunching of dumpers at the loading site is not permitted to avoid cooling of the hot asphalt mix. Similarly the availability of dumpers at the plant site has to be always ensured for the collection of hot asphalt mix. The plant output in tons per hour is dependent on plant capacity and varies in certain range during continuous operation. Hence the operation of number of dumpers in the system is to be optimized to avoid wastage due to excess queuing and idling of vehicles and eliminate any shortages. The queuing theory (Kalashnikov, 1990; Allen, 1990; Khanna, 2005) may be applied both at plant and laying site to find the optimized solution.

Optimization Problem
The optimization problem is to synchronize the plant production and laying operations by employing optimum number of dumpers and maintaining the plant output at its maximum capacity and minimizing the wastage of Hot Mix Asphalt due to excessive cooling during its transport from plant to laying site.

Constraints
The constraints in the present problem may be outlined as under:

- The plant production rate is maintained at highest level, but due to the variable factors such as moisture and temperature, the production rate is variable.
- The transport time is dependent on traffic, environmental factors, and average speed of the trucks, weighing, and covering time of the trucks. All these factors make the arrival rate at laying site more and more unpredictable.
- The service rate at laying site is dependent on capacity of paver which is constant but it varies due to variable mat thickness, compaction effort, etc.
- The arrival rate at plant site is again dependent on laying rate of pavers and traffic considerations for empty dumpers, average speed, lubrication of truck beds, etc.
Assumptions
Since there are uncertainties associated with the service and arrival rates at laying as well as HMA production site, the few assumptions have been made in consonance with the simple queuing model. The service and arrival rates at plant and laying site being variable, the following probability distributions may be assumed in the mathematical model.

• The arrival rates at the plant and laying sites may be assumed to have Poisson’s distribution. The service rates of dumpers at plant and laying sites may be assumed to have exponential distribution.

• Although the number of dumpers is finite in number, these are in constant circulation from Plant to laying site and back to plant. Hence the infinite customer population model of queuing theory may be assumed to be applicable in this case.

Notations
The following notations have been adopted in developing the model.

\( \lambda_p \): The average arrival rate of dumpers/h at the plant site assuming poison’s distribution.

\( \mu_p \): The average service rate of dumpers/h at plant site assuming exponential distribution.

\( \mu_l \): The average service rate at the laying site assuming the exponential distribution.

\( \lambda_l \): The average arrival rate at the laying site assuming poison’s distribution.

\( D_{ps} \): The average number of dumpers in system (waiting + being served) at the plant site.

\( D_{ls} \): The average number of dumpers at laying site.

\( T_{ls} \): The average time spent (Waiting time+ serving time) by a dumper in the system at the laying site in hrs.

\( P_o \): Plant output in tons/h

\( C_d \): Capacity of the dumper

\( S_d \): Average speed of the dumper in km/h

\( D_{TR} \): Number of dumpers in transit

\( D_{ADD} \): Additional dumpers to take care of breakdowns.

Distance: Average distance between plan and laying site in km.

\( D_{opt} \): Optimum number of dumpers in the system.

Hot Mix Plant Site
The arrival rate at the plant site may be assumed to have Poisson’s distribution. The service rate of dumpers at plant site may be assumed to have exponential distribution. Although the number of dumpers is finite in number, these are in constant circulation from plant to laying site and back to plant. Hence the infinite customer population model of queuing theory may be assumed to be applicable in this case. The average numbers of dumpers in the system (waiting + being served) at plant site is given by following formula as per queuing theory: (Khanna, 2005).

\[
D_{ps} = \frac{\lambda_p}{\mu_p - \lambda_p} \quad \ldots(1)
\]

The average service rate of dumpers/h at plant site assuming exponential distribution (Khanna, 2005) is

\[
\mu_p = \frac{P_o}{C_d} \quad \ldots(2)
\]

Hence from Equations (1) and (2), the average arrival rate of dumpers/h at the plant site assuming Poisson’s distribution (Khanna, 2005) is
\[ \lambda_p = D_{ps} \times \left( \frac{P_o/C_d}{1 + D_{ps}} \right) \]  

... (3)

The constraint is that there should always be at least one dumper in waiting at plant site to avoid any wastage of hot ready mix. Hence at least two dumpers should always be in system at the plant site. Also average service rate of plant depends upon its capacity with normal running.

**Laying/Paver Site**

At the laying site the requirement is that the bunching of dumpers should not occur and the waiting time for the dumpers should be minimum to avoid wastage due to excessive cooling of the hot ready mix. The arrival of dumpers is assumed to have Poisson distribution while the service rate is assumed to be exponentially distributed.

The average time a dumper spends on the system (waiting + being served) at the laying site is given by the following formula as per queuing theory. (Khanna, 2005) (Agrawal, 1985)

\[ T_{ls} = \frac{1}{\mu_l - \lambda_l} \]  

... (4)

Hence the average service rate of at the laying site

\[ \mu_l = \frac{(1 + \lambda_l \times T_{ls})}{(T_{ls})} \]  

... (5)

**Synchronization**

The synchronization requires that:

The arrival rate at laying site = service rate at the plant site since the dumpers are in circulation.

Hence

\[ \mu_p = \lambda_l \]  

... (6)

The average number of dumpers at laying site \( D_{ls} \) is calculated by the following queuing theory formula: (Khanna, 2005)

\[ D_{ls} = \frac{\lambda_l}{\mu_l - \lambda_l} \]  

... (7)

Putting \( \lambda_l = \mu_p = P_o/C_d \) and

\[ \mu_l = \frac{(1 + \lambda_l \times T_{ls})}{(T_{ls})} \]

\[ D_{ls} = (P_o/C_d)[\{(1+\lambda_l \times T_{ls})/(T_{ls})\} \]

\[ - (P_o/C_d)] \]  

... (8)

Optimized quantity of dumpers in the system.

With the above formulations, the total requirement of dumpers for the flexible pavement work execution supply chain can be optimized. The additional dumper for breakdowns can be catered based on practical judgment.

Hence the total number of optimized requirement of dumpers = Number of dumpers at plant site + Number of dumpers at laying site + Number of dumpers in transit + additional dumper to care of breakdowns

\[ D_{opt}=D_{ps}+D_{ls}+D_{TR}+D_{ADD} \]  

... (9)

Number of dumpers in transit can be calculated by following formula

\[ D_{TR} = 2 \times \text{Distance} \times \frac{\mu_p}{S_d} \]  

... (10)

Putting the values of \( D_{TR} \) and \( D_{ls} \) in the equation

\[ D_{opt} = D_{ps} + \left( P_o/C_d \right)[\{(1+ (P_o/C_d) x T_{ls})/(T_{ls})\} \]

\[ - (P_o/C_d)]+2 \times \text{Distance}\times P_o/C_d \times S_d \]

\[ + D_{ADD} \]  

... (11)

**Validation of the Formula**

The formula so developed has been validated by application to a practical case of construction of runway flexible pavement. The optimized quantity of dumpers to be deployed obtained by this model by changing various variables has been found correct on ground.
Maximizing the Plant Output in Synchronized System

The aim of the HMA production is to maximize the plant output without compromising the quality of asphalt mix and laying operation. If proper synchronization is maintained and the constant quality inspection is done at plant and laying site the quality aspect can be taken care of. Using the Equation (8) the plant output can be expressed in terms of the parameters such as number of dumpers at laying site, capacity of dumpers and the time a dumper spends at the laying site.

Hence,

\[ P_o = \frac{C_d \cdot D_{ls}}{T_{ls}} \] \hspace{1cm} \text{(12)}

The plant output can be maximized if the Dumper waiting time at laying site is minimum and capacity of the dumper is more. More is the number of dumpers which can be accommodated at the laying site more is the possibility of running the plant to its full capacity. This relationship may be useful in selection of right type and capacity of asphalt machinery at the design stage.

RESULTS AND DISCUSSION

The quantity of dumpers deployed for transportation of HMA may be calculated by using the formula developed by application of queuing theory. It has been ensured in the developing the formula (Equation 11) that the operational synchronization between plant production and laying site is maintained, the waiting time at the laying site is within limits so as to avoid excessive cooling of aggregates and the optimum number of dumpers at plant site is available.

The quantity of dumpers is dependent on variables such as plant output, average speed of the dumper, allowable waiting time at laying site, etc. The results obtained from ground data in construction of flexible pavement of a runway have been plotted in Figures 1, 2, 3 and 4. As it quite obvious in Figure 1 the requirement of dumpers have a direct relationship with the plant output. Since the aim is to run the plant to its full capacity the optimum number of dumpers obtained for 120 Tons/per h plant is 8.5 or 9 (Putting values of other parameters as shown in Figure 1).

Figure 2 depicts that the aim should be use higher capacity dumpers within the restrictions of the capacity of the paver finisher and HMA plant hopper. Figure 3 shows that the servicing time and waiting time at laying site should be reduced for better synchronization. The Figure 4 shows that the speed of the dumpers which depends on

\[ \text{Figure 1: Number of Dumpers Vs. Plant Output} \]

\[ \text{Figure 2: Number of Dumpers Vs. Dumper Capacity} \]
In addition to this, as per equation 12 the plant production can be maximized or a plant of higher capacity can be installed if the laying site can handle more numbers of large capacity dumpers for which the waiting and servicing time is minimum. These aspects can prove to be useful in selection the type of paver finisher, dumper capacity its type and the capacity of the plant in the construction of flexible pavement. If the dumper capacity is 12 tons and average number of dumpers is 2 at the laying site, a reduction of waiting and servicing time from 12 min to 10 min can allow the plant capacity to go up to 144 tons per h from 120 tons per h. A decision for a deployment material transfer vehicle at laying site and end bottom type dumpers can be taken depending upon the requirement of the HMA production. This aspect is depicted in Figure 5.

**CONCLUSION**

The optimized quantity of dumpers to be deployed in a synchronized system of Hot Mix Asphalt Plant and laying operations can be obtained by using queuing model. The results obtained from the model have been tested in a runway construction project and found correct. The application of the formula has resulted in eliminating the wastages due to deployment of too few or too much quantity of dumpers for the transportation of hot mix asphalt. The formula can be used by the construction engineers and site executives in day to day planning by putting the values of the variables. Also a relationship has been developed between the plant output, allowable number of dumpers at laying site, paver servicing capacity and dumper capacity. This relationship can be useful in the selection of asphalt mix machineries for the pavement construction.
REFERENCES


