EVALUATION OF DENOISING ALGORITHMS FOR TENSOR IMAGES BASED ON WESNR AND PARAFAC

G Kalaivani* and M Shobana

*Corresponding Author: G Kalaivani sakthigunasekaran@gmail.com

Hyperspectral images (HSI) is one of the tensor image which attract more and more interest in recent years in different fields, such as agriculture, military, and geography. They use the HSI to do the target detection or classification to find objects or minerals in the ground. Unfortunately, in the capturing procedure, the HSI is usually damaged by several types of noise such as signal independent noise and signal dependent noise are the noise present in hyper spectral image, removed by means of tensor based decomposition methods. The overall system comprises of three types of algorithm namely: parallel factor analysis (PARAFAC), hyper spectral noise estimation (HYNE) and multidimensional Wiener filter (MWF) with weighted encoding with sparre nonlocal regularization (WESNR). There are three steps, in the first step, the PARAFACSI-PARAFACSD method, uses a multi linear algebra model, PARAFAC decomposition, to remove SI and SD noise, respectively. In the second step the combination of multiple-linear-regression-based approach termed as the HYNE method and PARAFAC decomposition is used, which is named as the HYNE-PARAFAC method. The last one combines the MWF method and PARAFAC decomposition and is named as the MWF-PARAFAC method. First, PARAFAC decomposition, HYNE and MWF methods can be applied to remove the SI white noise, or PARAFAC decomposition and the HYNE method can be used to reduce the coloured SI noise. Then, to reduce the residual SD components, PARAFAC decomposition is applied to the HSI. The performance of the proposed PARAFAC SI-PARAFAC SD, HYNE-PARAFAC, and MWF-PARAFAC methods is validated on the simulated HSIs distorted by both SD and white SI noise.

Keywords: Hyperspectral image(HSI), PARAFAC, HYNE, MWF, WESNR, Signal dependent noise (SD), Signal independent noise(SI)

INTRODUCTION

A hyperspectral image (HSI) normally consists of hundreds of spectral bands and is also named as a tensor which provides much useful information in geography, agriculture, and military. For instance, Hyperspectral Digital Imagery Collection Experiment (HYDICE), can be represented as multidimensional data: two spatial...
dimensions and one spectral dimension. Acquired hyperspectral images are, in general, disturbed by the noises such as fixed pattern noise and random noise, which impairs the useful information and disturbs the scene interpretation. Photon and thermal noise are examples of random noise in HSI, whereas striping, periodic and interference noise are examples of fixed pattern noise which is generated by errors in the calibration process (Rakwatin et al., 2007) and can be removed by suitable procedure. Denoising is of great interest for target classification or detection (Renard and Bourennane, 2008) with the underlying principle of targets being distinguishable. As HSIs are normally produced by a series of sensors, the noise mainly comes from two aspects: signal-independent (SI) electronic noise and signal-dependent (SD) photonic noise. Therefore, denoising methods have become a critical step for improving the subsequent target detection and classification in remote sensing imaging applications.

The classical denoising methods rearrange the HSI into a matrix whose columns contain the spectral signatures of all the pixels. Matrix-based techniques cannot take advantage of spectra in hyperspectral image, Therefore, in order to treat the HSI as a whole entity, some new techniques were developed. For example, an HSI was treated as a hypercube in order to take into account the correlation among different bands (Liu et al., 2012) tensor-algebra was brought to jointly analyze the 3D HSI.

The newly proposed tensor-based denoising methods utilize multilinear algebra to analyze the HSI tensor directly. There are two important tensor decompositions: TUCKER3 and PARAFAC. These two decompositions play significant roles in analyzing tensors. Thus, a tensor decomposition method for instance Tucker3 model, is used to denoise HSI and permits to appreciate the denoising efficiency.

However, the decomposition by Tucker3 model is not unique, and it is difficult to estimate multiple ranks \( K = (K_1, K_2, K_3) \). The canonical decomposition/parallel factor analysis (PARAFAC) \( ^{[10]} \), considered as a higher order generalization of singular value decomposition and principal component analysis (PCA), relies on the rank-1 tensor decomposition to present data in a simple and restricted way. The distinguishing character of PARAFAC is its uniqueness property, that is to say that low-rank PARAFAC decomposition can be unique for rank values higher than one. Based on this property, we exploit the PARAFAC model to denoise HSI for the first time and improve classification results. First, we compute the optimal rank of PARAFAC denoising, i.e the best effectiveness of denoising can be obtained by PARAFAC decomposition at the optimal rank. By assuming that the noise is HSI is spectrally uncorrelated and that the signal has strong spectral correlation. The noise model parameters were estimated by multiple linear regressions (MLR) in, which is herein after abbreviate as the HYNE method. Since SD noise depends on the signal, with such an assumption, a considerable part of the removed noise is SI. According to the different statistical properties of SI and SD noise, in the proposed system three two-step methods to remove these two types of noise are used.

First SI noise is reduced by PARAFAC decomposition, the HYNE method, or the MWF method. Then, the residual SD components can be further reduced by PARAFAC decomposition due to the statistical property of SD noise. The proposed hybrid method: the PARAFAC-SI–PARAFACSD method, which uses PARAFAC decomposition twice the HYNE-PARAFAC
method, which is a combination of the HYNE method and PARAFAC decomposition. The MWF-PARAFAC method, which combines the MWF method and PARAFAC decomposition. If the SI noise is colored, then the HYNE method and PARAFAC decomposition can effectively remove this colored SI noise.

The scatter-plot method is used for the noise correlation coefficients estimation. The proposed system focus only on the HSIs, distorted by both SD and white or colored SI noise. The experimental results show that the proposed methods are efficient in the reduction of both SI and SD noise in HSIs.

BRIEF REVIEW ABOUT HSI AND TENSORS

A. Spectral Image Analysis

To understand the advantages of hyperspectral imagery, it may help to first review some basic spectral remote sensing concepts. You may recall that each photon of light has a wavelength determined by its energy level. Light and other forms of electromagnetic radiation are commonly described in terms of their wavelengths. For example, visible light has wavelengths between 0.4 and 0.7 microns, while radio waves have wavelengths greater than about 30 cm (Figure 1). Reflectance is the percentage of the light hitting a material that is then reflected by that material (as opposed to being absorbed or transmitted). Some materials will reflect certain wavelengths of light, while other materials will absorb the same wavelengths. These patterns of reflectance and absorption across wavelengths can uniquely identify certain materials.

B. Imaging Techniques

Depending on the number of spectral bands and wavelengths measured, an image is classified as a multispectral image when several wavelengths are measured and a hyperspectral image when a complete wavelength region, i.e., the whole spectrum, is measured for each spatial point. For example, a RGB image from a typical digital camera is a type of multispectral image that uses the light intensity at three specific wavelengths: red, green, and blue, to create an image in the visible region. The Figure 2 compares the optical information obtained by monochrome cameras, RGB cameras, and hyperspectral cameras.

Figure 1: Electromagnetic Spectrum

Figure 2: Differences in Imaging

This article can be downloaded from https://www.ijerst.com/National-Conference-on-RTCIT-2015.php#1
C. Imaging Spectrometer

Hyperspectral images are produced by instruments called imaging spectrometers. The development of these complex sensors has involved the convergence of two related but distinct technologies: spectroscopy and the remote imaging of Earth and planetary surfaces. Imaging spectrometers measure the light reflected from many adjacent areas on the Earth’s surface. In many digital images, sequential measurements of small areas are made in a consistent geometric pattern as the sensor platform moves and subsequent processing is required to assemble them into an image.

D. Tensor

A tensor, represented as \( Y \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N} \) is defined as a multidimensional array which is the higher-order equivalent of the vector (one-order tensor) and a matrix (two-order tensor). In this study, the HSI data cube is regarded as a three-order tensor \( Y \in \mathbb{R}^{I_1 \times I_2 \times I_3} \) in which modes 1 and 2 represent the spatial modes and mode 3 denotes the spectral mode. Taking each vector to be in different mode, we can visualize the outer product of three vectors as follows:

Mathematically, we can write the outer product of three vectors \( a; b; c \) as follows,

\[
\begin{pmatrix}
a_1 \\
a_2 \\
\end{pmatrix} \otimes \begin{pmatrix}
b_1 \\
b_2 \\
\end{pmatrix} \otimes \begin{pmatrix}
c_1 \\
c_2 \\
\end{pmatrix} = \begin{pmatrix}
\alpha_{a} b_{c} c_2 \\
\alpha_{a} b_{c} c_2 \\
\end{pmatrix} \alpha_{b} c_{c} c_2 \\
\alpha_{b} c_{c} c_2 \\
\end{pmatrix}

We can see that the indexes of the entries in the resulting tensor. Tensor matricization reorders the elements of an N-order tensor into a matrix from a given mode. The n-mode matricization of \( X \) belongs to \( \mathbb{R}^{L_1 \times L_2 \times \ldots \times L_N} \) is matn\( X \) belongs...
to $RL_i\times(L_1L_2...\text{Ln}-1\text{ Ln}+1...\text{LN})$, which is the ensemble of vectors in the n-mode obtained by keeping index $L_i$ fixed and varying the other indices. A visual illustration of tensor matricization is shown in Figure 5.

Figure 5: Illustration of Tensor Matricization of Three Modes

**METHODOLOGY**

The given HSI is taken as input to the system. This HSI image is read and displayed. Then the HSI image processed in the HYNE algorithm to provide the Rank-1 Tensor profiles. With these profiles, we perform the Alternative Least Square Algorithm to optimize the tensors. Then we sort the tensors of higher order and reconstruct the noise free image by combining signal dominant components.

**A. HSI Image Reader**

The Hyperspectral imaging (HSI) collects and process information from across the electromagnetic spectrum. Much as the human eye sees visible light in three bands (red, blue, green), spectral imaging divides the spectrum into many more bands. This technique of dividing images into bands can be extended before can be extended beyond the visibility. Hyperspectral sensors collect information as a set of ‘images’. Each image represents a range of the electromagnetic spectrum and is also known as a spectral band. These ‘images’ are then combined and form a three dimensional hyperspectral data cube for processing and analysis. This module is designed to read and visualize the HSI images.

The HSI data is considered as multiple images combined as a cube. Thus we have view each image in a well furnished manner. Each image has slice of images of different colors. This slice of image is not taken as a single color image for the calculation instead it is taken as whole cube called tensors. By extending the data model in [8] to the 3-D representation, a noisy HSI can be expressed as a third-order tensor $R \in R^{n \times 2 \times 3}$ composed of a multidimensional signal $X \in R^{n \times 2 \times 3}$ impaired by an additive random noise $N(X)$

$$R = X + N(X) \quad \text{(1)}$$

where $N(X)$ accounts for both SI and SD noise, and its variance depends on the pixel $x_{i1,i2,i3}$ in the useful signal $X$ Elementwise, the data model is [8]

$$r_{i1,i2,i3} = x_{i1,i2,i3} + \sqrt{x_{i1,i2,i3}} u_{i1,i2,i3} + w_{i1,i2,i3} \quad \text{(2)}$$

where $u_{i1,i2,i3}$ is a stationary zero-mean uncorrelated random process independent of $x_{i1,i2,i3}$ with variance $\alpha_2 u_{i3}$ and $w_{i1,i2,i3}$ is electronics noise, which is zero-mean WGN in each band with variance $\alpha_2 w_{i3}$. The additive term $x u$ is the generalized SD noise and denoted as photonic noise, and $w$ is the SI noise component and is generally assumed to be Gaussian distribution in each band. Then, we can define $N(X) = NSD(X) + NSI = NSD(X) + W$, and
(1) can be correspondingly rewritten as
\[ R = X + \text{NSD}(X) + W. \]  

With the assumption that \( x, u, \) and \( w \) are independent, and both \( u \) and \( w \) are zero mean and stationary, the noise variance of each entry \( n(X)_{i1,i2,i3} = \sqrt{x_{i1,i2,i3}} u_{i1,i2,i3} + w_{i1,i2,i3} \) of \( N(X) \) can be written as [5], [7], [8], [29]
\[ \sigma^2_n(X)_{i1,i2,i3} = \sigma^2_x \sigma^2_u + \sigma^2_w \]  

The unfolding matrix \( R_3 \in \mathbb{R}^{I_3 \times M_3} \) of the HSI data tensor \( R \in \mathbb{R}^{I_1 \times I_2 \times I_3} \) can be expressed as
\[ R_3 = X_3 + N(X)_3 \]  

where \( X_3 \) is the 3-mode unfolding matrix of the multidimensional signal tensor \( X \) and
\[ N(X)_3 = \text{NSD}(X)_3 + W \]  

with \( \text{NSD}(X)_3 \) and \( W_3 \) being the 3-mode unfolding matrices of \( \text{NSD}(X) \) and \( W \), respectively. Using the mean noise variance of the \( i \)th spectral band the covariance matrix of the 3-mode unfolding matrix \( W_3 \) of the SI noise tensor \( W \) can be written as a diagonal matrix, i.e.,
\[ C^{(3)}_W = \text{diag} (\sigma^2 w_1, \sigma^2 w_2, ..., \sigma^2 w_{I_3}) \]  

Based on the assumption of the independence of \( x \) and \( u \), where \( u \) is zero-mean and independent between spectral bands, the covariance matrix of the 3-mode unfolding matrix \( \text{NSD}(X)_3 \) can be expressed as
\[ C^{(3)} \text{NSD}(X) = \text{diag} (\mu_1 \sigma^2 u_1, \mu_2 \sigma^2 u_2, ..., \mu_{I_3} \sigma^2 u_{I_3}) \]  

**NOISE REDUCTION**

With the advances in the technology of optoelectronics imaging devices, the SD photonic noise contribution has become as dominant as the SI electronic noise in HSI data. Hence, preprocessing and analysis methods must be revised or designed to take into account the SD noise. Since the HYNE and MWF methods and PARAFAC decomposition were proved to be effective methods for denoising HSIs [5], [15], [16], [20], by using the widely accepted noise model in (4), we propose three two-step methods to first delete SI noise from HSI data \( R \) by the HYNE and MWF methods or PARAFAC decomposition and then reduce the SD components by PARAFAC decomposition according to the different statistical properties of the two types of noise.

**Figure 6: Proposed Algorithm for Reduction of SI and SD Noise**

**A. SI Noise Reduction**

The SI noise is generally assumed as zero-mean WGN whose covariance matrix is a scalar matrix with all its main diagonal entries being equal to the noise variance; hence, HYNE and MWF with WESNR methods can be used to denoise the WGN. PARAFAC decomposition can also denoise the WGN by selecting an appropriate rank [12], which is named as \( K_{SI} \). For the zero-mean
WGN, the covariance matrix \( C^{(n)}_{W} \) of the \( n \)-mode unfolding matrix of the removed noise \( \hat{W} = R^{-1}R \) should approach a scalar matrix \( \sigma_2w1\ln \) with \( \sigma_2w \) being the SI noise variance, \( \ln \) being an identity matrix of size \( ln \), and \( n = 1, 2, 3 \). If the values of the diagonal elements \( c_{in}, \in \) of \( C^{(n)}_{W} \) approximate equally, that is to say, that the variance of the diagonal elements of \( C(n) \hat{W} \) is small enough, and the squared norm of the covariance \( ||C(n)\hat{W}||^2 \) is also quite close to the sum of the diagonal elements.

Then this \( C^{(n)}_{W} \) can be considered approaching a scalar matrix. This criterion can be used to estimate the rank \( KSI \) of PARAFAC decomposition for the reduction of SI noise. However, in some HSIs, the SI noise was colored i.e., non-white. In this case, as mentioned above, the HYNE method and PARAFAC decomposition can also effectively reduce the colored SI noise. For the PARAFAC decomposition to remove the colored SI noise, it is still necessary to select the appropriate rank \( KSI \). Since the variance of the colored SI noise is different from band to band, the covariance matrix \( C^{(3)}_{W} \) of the 3-mode unfolding matrix of the removed noise \( \hat{W} = R^{-1}R \) should approach a diagonal matrix \( \text{diag}(\sigma_2w_{1}, \sigma_2w_{2}, \ldots, \sigma_2w_{3}) \), where \( \epsilon \sigma_2w_{i} \mu_{3} \) being the variance of SI noise in the \( i \)-band of HSI and \( \mu_{3} = 1, \ldots, \mu_{3} \). If the squared norm of the covariance \( ||C^{(3)}_{W}||^2 \) is quite close to the sum of the diagonal elements, then this \( C^{(3)}_{W} \) can be considered approaching a diagonal matrix. This criterion can be used to estimate the rank \( KSI \) of PARAFAC decomposition for the reduction of colored SI noise. Thus, most of the SI noise can be removed by the HYNE and MWF methods or PARAFAC decomposition while a large number of SD components are remained in the estimated tensor \( \hat{R} \) due to its dependence character on signal \( X \).

According to the results of noise reduction by the HYNE method in [5], the MWF method and PARAFAC decomposition in [12], the influence on the signal by the filtering processes is very little and can be ignored. Therefore, in this paper, we assume that the statistical properties of both the signal and the remaining SD noise after the SI noise reduction are relatively constant in the estimated tensor \( \hat{R} \) and the covariance matrix of the SD noise in (11) can still be expressed by \( C^{(3)}_{NSD,X} = \text{diag}(\mu_{1}, \sigma_2u_{1}, \mu_{2}, \sigma_2u_{2}, \ldots, \mu_{3}, \sigma_2u_{3}, \mu_{3}) \), where \( \epsilon \sigma_2u_{i} \mu_{3} \) and \( \epsilon \sigma_2u_{i} \mu_{3} \) are the variance of random process \( u_{1}, u_{2}, \mu_{3} \) and the mean of all pixels in the \( 3 \)-th band of \( \hat{R} \), respectively.

### B. SD Noise Reduction

Since SD noise is the dominant part in the estimated \( \hat{R} \), here, we can neglect the remaining SI components after the denoising above. We know that the selection of the rank-\( KSD \) is the key factor to remove SD noise by PARAFAC decomposition, which can be done by the four steps of SD noise reduction described below. After the reduction of SI noise, assuming the SD noise is dominant in \( \hat{R} \), then each diagonal entry of the covariance matrix of the SD noise in (11) can be expressed as \( \sigma_2N(\chi, \mu_{3}) \approx \epsilon \sigma_2u_{i} \mu_{3} \). It is clear that \( \sigma_2u_{i} \mu_{3} \mu_{3} \mu_{1} = \epsilon \sigma_2u_{i} \mu_{3} \mu_{3} \mu_{1} \mu_{3} \), that is to say, that by multiplying the inverse \( \mu_{3} \) (\( \mu_{3} = 1, \ldots, \mu_{3} \)), the covariance matrix of SD noise becomes a diagonal matrix \( \text{diag}(\sigma_2u_{1}, \sigma_2u_{2}, \ldots, \sigma_2u_{3}) \). The criterion above to assess a matrix approaching a diagonal one can also be used here to Figure 6. Proposed algorithms for the reduction of both SI and SD noise.

Estimate the SD noise. Therefore, the reduction of SD noise by PARAFAC decomposition consists of the following steps.
1) Unfold the estimated noise tensor to \( n \)-mode unfolding matrix with \( n = 1, 2, 3 \).

2) Calculate the covariance matrix of the \( n \)-mode unfolding matrix of the estimated noise tensor.

3) Multiply the covariance matrix by \( \text{diag}(\hat{\mu} - 11, \ldots, \hat{\mu} - 13) \), i.e., the inverse of the mean value of each band of the noisy data \( \hat{R} \).

4) Use the criterion above to assess the result matrix obtained from step 3) approaching a diagonal matrix. Then, the rank \( KSD \) of PARAFAC decomposition for the reduction of SD noise can be obtained from these four steps.

**EXPERIMENTAL RESULT**

The proposed algorithm is applied in the HSI data. The HIS cannot be taken as an image itself. The values are to be plotted as an image for our visualization. Thus a set of values of the received image is plotted as an image for our visualization. The values are plotted as an image for the original data and for the Denoised data. The original values are not plotted fully, only certain area shown for the visualization for a clear idea of the HSI image. The three images with their denoised output is shown in Figures 7-8.

<table>
<thead>
<tr>
<th></th>
<th>Noisy Image</th>
<th>Noise Removed Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR</td>
<td>17.19</td>
<td>25.35</td>
</tr>
<tr>
<td>SNR</td>
<td>5.3</td>
<td>32.60</td>
</tr>
<tr>
<td>MSE</td>
<td>1263.75</td>
<td>180.37</td>
</tr>
</tbody>
</table>

**CONCLUSION**

The HSI noise is removed using the tensor based decomposition method such as PARAFAC, HYNE and MWF with WESNR. The PARAFAC, HYNE and MWF with WESNR are used for removing the SI noise in the image and Reduction of the SD noise is carried out using PARAFAC decomposition. And HYNE. The SNR, PSNR and MSE values 5.3, 17.19 and 1263.75 respectively for the input noisy image were been denoised and the image has achieved high SNR=32.60, PSNR=25.3575 and low MSE=189.37 value. Thus the proposed system reduces the noise greatly.

**REFERENCES**


